NUCLEAR FORENSICS OF THE B8: HEISENBERG'S LAST REACTOR

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ABSTRACT

Reconstructing the early history of nuclear programs provides a unique opportunity to practice and understand the limits of nuclear forensics methods. This paper reconstructs and analyzes the B8, the last reactor experiment by the German nuclear program in World War II. While it was not successful in achieving criticality, its designer Werner Heisenberg claimed in a 1947 *Nature* article that he would have needed just a "small amount" more natural uranium and "not quite half" more heavy water. Our study digs through various archives to rediscover various original specifications of the B8. Our Monte Carlo simulations reveal that the B8, as built in April 1945 with 664 natural uranium cubes and 1,400 L of 96.8% D_2O, achieved a criticality of $k_{\text{eff}} = 0.93782 \pm 0.00011$. At a minimum, the B8 would have required 1,500 cubes and 3,390 L of D_2O. This reconstruction not only enhances understanding of historical reactor design but also informs modern nuclear forensics and nonproliferation efforts.

Keywords document-based nuclear archaeology, graphite, Uranverein, Heisenberg, Bothe, Hanle

1 Introduction

The study of historical reactors not only enhances our understanding of early nuclear science but also allows us to challenge the limits of nuclear forensics methods. For instance, while the Manhattan Project is well-documented, studying the concurrent German program can unravel the complexities of interpreting past nuclear projects. Meticulously reconstructing and analyzing the B8 reactor, built in the final days of World War II, helps establish a detailed technical lineage that aids in the identification and attribution of nuclear materials, ensuring that lessons from the past are integrated into modern nuclear security frameworks.

The German Army had established a testing center[2] for nuclear experiments in Gottow under physicist Kurt Diebner and General Karl H. E. Becker, akin to the Los Alamos site under Oppenheimer and General Groves. However, Diebner had only been a government consultant before the war and lacked the academic clout to really consolidate institutions and physicists under his authority. This allowed Nobel-winning physicist Werner Heisenberg to receive part of the Army's nuclear materials and pursue pile experiments at the University of Leipzig. Animosity festered between the two, deepening after Heisenberg replaced Diebner's co-appointment as acting director to the prestigious Kaiser Wilhelm Institute for Physics at Berlin in July 1942; Heisenberg continued his experiments at newly built facilities there while Diebner tested his own piles at Gottow. The various piles attempted at Gottow, Leipzig, and Berlin would be numbered, with suffixes "G," "L," and "B" respectively, e.g., G3, L4, B8.

By late 1944, to escape the Allied bombings of German cities, the Heisenberg and Diebner consolidated the Uranverein's uranium and heavy water in the city of Stadtilm. At Heisenberg's insistence, his KWIP team then made off by

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[2]Formal name: *Chemisch-physikalische- und Atom-Versuchsstelle Gottow* (Chemical-Physical and Atomic Testing Station Gottow), part of *Heeresversuchsanstalt Kummersdorf* (Army Research Center Kummersdorf) under *Wa Prüf 11 Abteilung Sondergerät* (Army Development and Testing Group 11, Department for Special Equipment).

foot, bike, and cart to the town of Haigerloch in the Black Forest taking with them the majority of uranium and about half the heavy water. There in Haigerloch, Heisenberg would conduct his last reactor experiment—the B8.

Built in April 1945 a month before Germany's surrender, B8 reactor[3] was the last of the Uranverein's reactor experiments. The B8 had a cylindrical core of radius 64 cm and height 162 cm with 664 natural uranium fuel cubes suspended as alternating chains of 9 and 10 cubes in 5 concentric rings, filled by 1,400 L of heavy water moderator. This core was surrounded by natural graphite on all sides, and then again by light water 60 cm on all sides. Although the B8 did not achieve effective criticality, it was the closest of all German attempts, with Heisenberg and physicist Karl Wirtz measuring an infinite multiplication $k_\infty = 1.11$ in-situ.

While the original Uranverein report on the B8 is lost, we are guided by two primary sources written by Heisenberg. Regarding the B8, Heisenberg wrote in a 1947 *Nature* paper:

> The material available at Haigerloch, however, was just insufficient to attain [criticality]. A relatively small amount of uranium would in all probability have sufficed; but it was no longer possible to obtain it, since transport from Berlin or Stadtilm could no longer reach Hechingen [closest town to Haigerloch]. [1]

He also wrote in the *Fiat Review of German Science* (1948):

> As can be seen from the measured values $Z = 6.7$... Here the Haigerloch arrangement is considered as a sphere of radius 71 cm. The critical point $Z \to \infty$ would therefore have been reached at radius ≈ 80 cm which corresponds to an increase of the volume by not quite half. In practice, however, the direct enlargement was not planned, since more D_2O was not available, but U-pieces would also be brought into the graphite jacket, which would probably have been sufficient to reach the critical point. [2]

The B8 used 1,400 L of heavy water; Heisenberg is claiming a "small amount" more uranium and about 2,100 L of heavy water would have been all that was needed for criticality. To be clear, this claim is immediately skeptical compared to the material footprint of Chicago Pile 3, which was a similar natural uranium, heavy water reactor built by the Manhattan Project in 1944. CP-3 used 5,678 L of heavy water and 2,558 kg of natural uranium, over twice that of Heisenberg's expectations.

Since, then there have been a few modern analyses of the B8, namely in 2009 by an Italian team and in 2019 by a Serbian physicist [3; 4]. The Italian team claimed Heisenberg recorded the B8 to have $k_{\text{eff}} = 0.85$ and to have replicated this result; the Serbian paper contests this result as improbable considering $k_\infty = 1.11$ and that it was probably obtained from modeling errors. Both papers make several assumptions on the purity of the B8's materials.

In this paper, we wish to address the following questions about the B8:

1. By faithfully reconstructing its material specifications, what was the criticality of the B8 was it was built in April 1945?
2. Would the B8 have gone critical with "a relatively small amount of uranium" and/or "an increase of the [heavy water] volume by not quite half," as Heisenberg claimed in 1947–48?
3. If not, what would be the minimum materials and size necessary for the B8 to achieve criticality, $k_{\text{eff}} \geq 1$?

A roadmap is as follows: In Section 2, we present a basic qualitative overview of the nuclear processes under consideration in reactor design and optimization. In Section 4, we show the Italian claim that the B8 had $k_{\text{eff}} = 0.85$ was due to a translation error of Heisenberg's 1948 *Fiat Rev.* article. In Section 5 we model the B8 specifications into our neutronics simulation codes. Section 6 highlights the neutronic features and behaviors of the B8 as it was built ("fiducial B8") while Section 7 optimizes the B8 to find its minimum critical size and materials. The overall paper is written to be approachable to a general audience, so many calculation specifics have been included in the Appendix.

2 Nuclear Theory

Before delving into the B8, we find it convenient to qualitatively establish the basic considerations of reactor design.

A reactor can be described by its criticality, quantified by the *effective* multiplication factor k_{eff}. The fission reaction is kick-started with a steady stream of neutrons from a source like RaBe. Then, k_{eff} quantifies the ratio at which this initial population multiplies. A system with $k_{\text{eff}} < 1$ is subcritical; $k_{\text{eff}} = 1$ is critical, signifying a self-sustaining

[3] B8 is alternatively stylized as B_8, B_{VIII}, or B-VIII.

fission reaction; $k_{\text{eff}} > 1$ is supercritical. Fermi's Chicago Pile 1 in 1942 and Heisenberg's B8 in 1945 were both attempts to build a $k_{\text{eff}} \geq 1$ reactor. Fermi achieved $k_{\text{eff}} \approx 1.0006$ at CP-1.

The *infinite* multiplication factor k_∞ measures the neutron multiplication in an infinite medium, i.e., an infinitely large reactor with no leakage out the physical surface area. The intent of k_∞ is to understand the inherent neutron economy of the reactor fuel and moderator configuration itself. Once $k_\infty > 1$, i.e., the reactor can theoretically sustain a chain reaction in an infinite medium, then we can worry about minimizing its leakage.

Uranium-235 fission requires much slower neutrons than it produces. Each neutron from fission has an average speed of 2 MeV. However, fission is a probabilistic process, and uranium-235 is more likely to fission with neutrons in the thermal range (< 1 eV). This slowing down, or thermalization, of fission neutrons is accomplished via elastic scattering with the reactor moderator. Heavy water (D_2O), graphite (C), and light water (H_2O) are good candidates, but they absorb neutrons in increasing order (and inversely to their cost). Especially these early reactors have very small neutron surpluses and are thus very sensitive to the moderator purity. Thus, the moderator must be carefully selected and purified. The Manhattan Project funded and succeeded with all three designs, the Germans elected for heavy water moderation due to its availability from occupied Norway, c.f. Ref.[5].

3 B8 Specifications

The B8 had a right cylindrical core of radius 64 cm and height 162 cm with 664 natural uranium cubes suspended as alternating chains of 8 and 9 cubes in 5 concentric rings [3]. The remaining volume was filled by 1400 L or 1549 kg of heavy water moderator. It is possible the core was not filled to the brim with heavy water in practice, although this would only reduce criticality. This core was surrounded by graphite and then by 60 cm of light water on all sides. The Atomkeller Museum, built on the exact site of the B8, informed us that the ambient temperature was 10°C, so all materials properties, e.g., density, cross sections, were evaluated at that temperature.

Each of the square 664 uranium cubes were cut to length 5 cm. The cuts are visually irregular and jagged, and small notches were made on the edges to fit the aluminum wire that tied each chain together. We picked[4] a mass density of each cube to be 18.5349 g/cm^3 due to internal porosity. We also modeled in an average impurity of 7.1 ppm by mass of equivalent boron content (EBC) based on analysis by by Mayer et al. [6].

The exact purity of the B8's heavy water was not recorded in any primary sources known to us. However, original samples from the B8 were apparently collected by the Alsos Mission and analyzed at the U.S. National Bureau of Standards in 1947; chemist E. R. Smith measured this purity to be 96.8 mol%. These tanks remain on display at the now-named U.S. National Institute of Standards and Technology.

The graphite reflector was built with $5\times10\times50$ cm blocks of Siemens electrographite of density 1.7 g/cm^3 [2]. Because arranging rectangles into a circle causes a "squaring the circle" problem, the graphite reflector had a bulk density of 1.58 g/cm^3 on the top and bottom, as calculated by Pešić [4]. Despite being "the purest graphite available in industrial quantities" in Germany at the time, Siemens electrographite had a thermal capture cross section of 12.2 mb, as benchmarked by Park and Herzele [5]. In contrast, American graphites before purification had average 6.2 mb of capture, which was brought down to 5.0 mb in purified AGOT graphite for Chicago Pile 1.

4 Prior Works

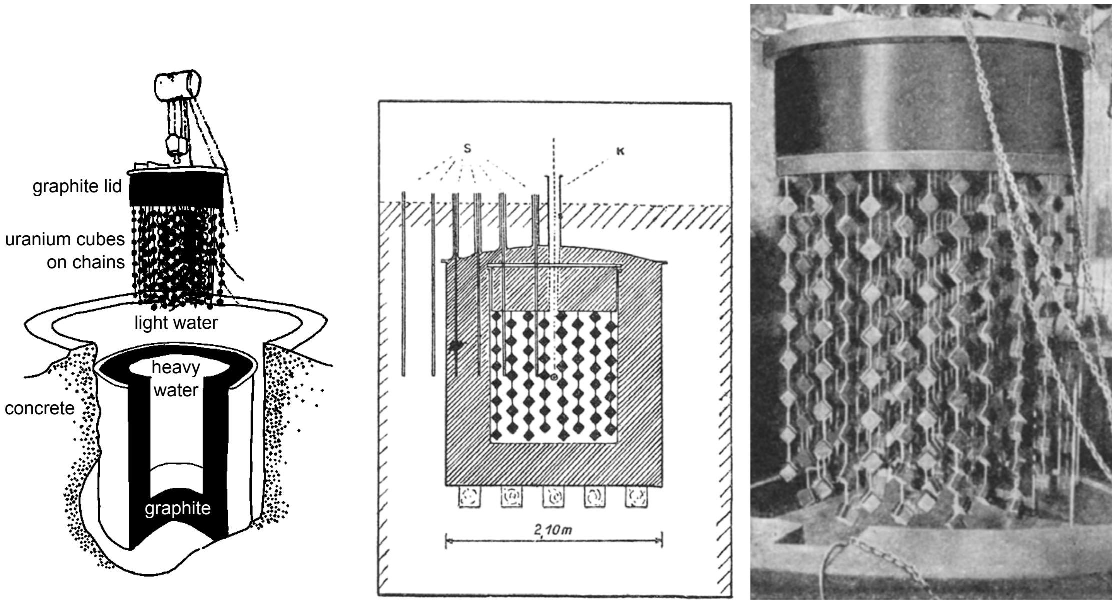
4.1 Heisenberg and Wirtz (1948)

After the war, several German scientists were requested to describe their wartime scientific activities in the American *Fiat Review of German Science* (albeit in German). Here, Heisenberg and B8 co-designer Karl Wirtz give a detailed overview of all the Leipzig and Berlin experiments, including refactoring all their wartime results to the modern convention of k_∞, giving $k_\infty = 1.11$ for the B8 [1], which Wirtz upholds in another 1989 note [7]. An English translation of this *Fiat Rev.* paper can be requested from the first author.

4.2 Grasso et al. (2009)

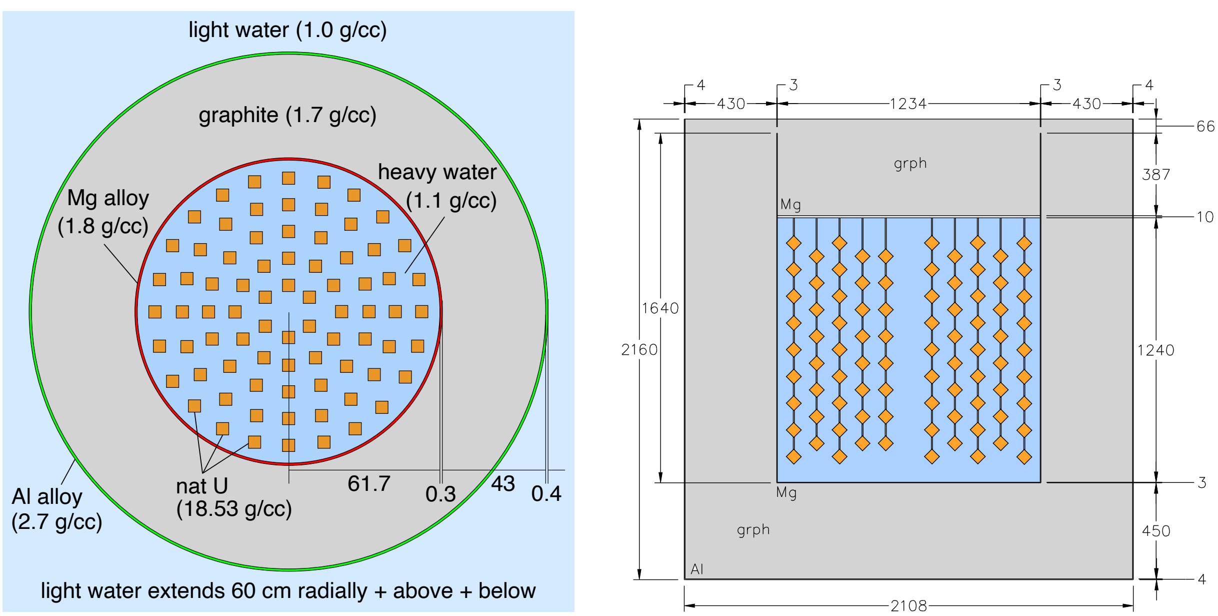
Grasso et al. were the first to technically evaluate the B8 [3]. They reconstructed the pile using assumed specifications of 19.05 g/cm^3 for uranium density and 95% purity for heavy water and tested the impact of graphite purity as a reflector. They claim to reproduce the k_{eff} of 0.85 that Heisenberg ostensibly measured in 1945. Grasso et al.

[4] Based on communication of results to be further detailed in a separate work.



(a) Isometric view. (b) Heisenberg's sketch [2]. (c) Original photo [3].

Figure 1: Various representations of the B8.



(a) Plan view. (b) Elevation view.

Figure 2: Annotated dimensions of the B8, in cm, as we modeled in MCNP6.2.

concluded that the lattice configuration needed to be spread further apart since the slowing down length in heavy water is about 11 cm, while the distance between fuel cube surfaces in the reactor was only 5 to 8 cm.

We believe there are two errors in Grasso et al.'s work:

1. Grasso et al.'s MCNP model simplifies the chains of cubes into homogeneous right cylinders. When the fuel and moderator are discretely modeled, like in a "real" reactor, the thickness of the fuel causes extra fast fissions, such that the outer portion of the fuel self-shields the interior. This decreasing thermal fissions but also prevents neutrons from being parasitically absorbed by uranium-238 resonances at intermediate (keV-range) energies. Thus, the net effect is that a real heterogeneous reactor has higher k than when its fuel is modeled homogeneously; Grasso et al.'s model would be underestimating k [8].

2. Grasso et al. claim that the B8 had $k_{\text{eff}} = 0.85$ because Heisenberg ostensibly recorded a "neutron-multiplication factor M of 6.7," which Grasso then converts to k_{eff} by:

$$k_{\text{eff}} = 1 - \frac{1}{M} = 1 - \frac{1}{6.7} \approx 0.8507. \tag{1}$$

The subcritical multiplication factor M defined in Eq.(1), is a commonly used variable in modern nuclear engineering. It represents the ratio of neutrons produced by the reactor source, which initiates the chain reaction, to the overall neutron production rate in the reactor. For example, if $M = 6.7$ and the B8's RaBe neutron source produced x neutrons per second, then $6.7x$ neutrons per second were generated in the reactor overall. As the reactor nears criticality ($k > 1$), the self-sustaining fission keeps going, and M approaches infinity.

However, what Heisenberg actually recorded in Ref.[2] was "multiplication factor" $Z = 6.7$, with Z defined as the ratio of the number of neutrons just outside the graphite reflector N_a vs. outside the outermost light water vessel N_0:

$$Z = \frac{N_a}{N_0} = 6.7. \tag{2}$$

Z also approaches infinity at $k > 1$ because the neutron population near the reflector grows faster than outside the vessel, but unlike M, the denominator of Z does not stay constant. So, Z and M measure different quantities and cannot be interchanged to calculate k_{eff}.

4.3 Pešić (2019)

Pešić models the real geometry of the B8 [4]. He incorporates 7.1 ppm EBC of impurities from Mayer's analysis into natural uranium fuel at an assumed density of 19.05 g/cm^3. He calculates a few possible arrangements of the Heisenberg's $5 \times 10 \times 50$ cm graphite blocks, determining the bulk reflector density to be 1.7 g/cm^3 on the sides and 1.58 g/cm^3 on the top and bottom. He then tests criticality with permutations of heavy water from 88% to 99% pure. He also contests Grasso's conclusion of $k_{\text{eff}} = 0.85$; reproducing Grasso's homogenized fuel model does obtain that value but undoing the homogenization and modeling the real geometry yielded a k_{eff} of 0.945.

5 MCNP Models

Three different MCNP models of the B8 was modeled for our calculations.

1. The **fiducial model** is the B8 reactor faithfully modeled as-built in April 1945, as shown in Figure 2.

2. The **unit cell model** is a Wigner-Seitz cell of a single fuel cube and its surrounding heavy water with periodic boundary conditions to simulate an infinitely large reactor, shown in Figure 3. An infinite reactor allows intrinsic nuclear properties of the fuel-moderator mix to be measured without confounding effects from neutron leakage. The dimensions of the unit cell are computed from the Voronoi decompositions of the fuel lattice in the fiducial model. The Voronoi decomposition tells us all the points in space that are closer to this specific fuel cube than to any other cube. Using the Python `scipy` package, the average Voronoi volume was 1787.75 cm^3 and surface area in the XY plane (plan view) 138.82 cm^2. Thus the unit cell has radius 6.65 cm and height 12.88 cm.

3. The **lattice model** is the fiducial model but with the fuel arranged in a hypothetical hexagonal close-packed lattice. The reasoning is discussed Section 7. The HCP lattice is created by imagining each cube to be circumscribed by a sphere and solving a sphere-packing problem. Since it is a regular 3D lattice, we wrote a Python script to obtain the HCP vertices by forming a line of equidistant points and then tessellating them into $\mathbb{R}^3$ with the linear transformation matrix:

$$\begin{bmatrix} 1 & 1/2 & 1/2 \\ 0 & \sqrt{3}/2 & \sqrt{3}/6 \\ 0 & 0 & \sqrt{2/3} \end{bmatrix}. \tag{3}$$

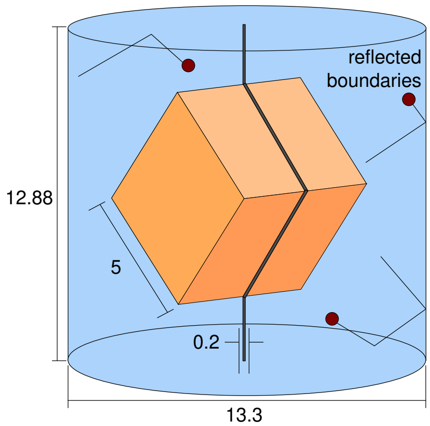


Figure 3: Wigner-Seitz unit cell of a single fuel cube in heavy water with reflected boundaries to simulate an infinitely large reactor; dimensions in cm.

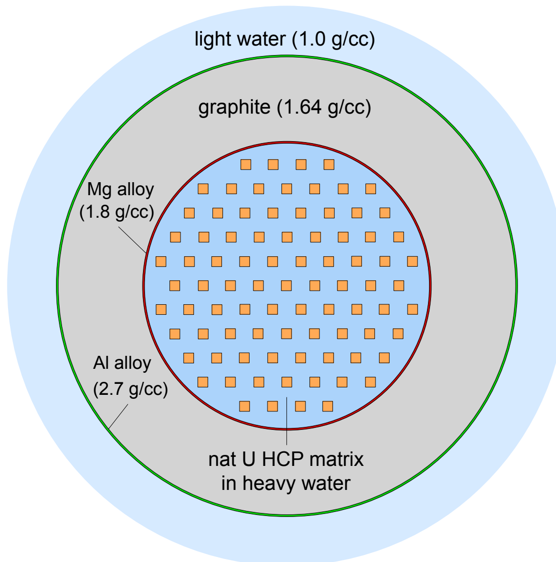


Figure 4: Plan view of the lattice model with fuel cubes arranged in a HCP lattice per Eq.(3).

We sorted the cube centers by $\texttt{max}\left(\sqrt{\texttt{x}^2+\texttt{y}^2}, |\texttt{z}|\right)$ which symbolically corresponds to the L_∞ norm of the radius and half-height in the proportions of the B8 reactor ($h \approx 2r$). In order to construct an N-cube cylindrical HCP lattice model, we took the first N cube centers from the sorted list and scaled the coordinates such that each cube center is at least 4.5 cm away from the reactor boundaries.

6 Neutronics Benchmark and Analysis

All calculations were run in MCNP6.2 using ENDF/B-VIII.0 libraries with temperature interpolation of cross sections[5] via pseudomaterials in the data cards. Because MCNP only has cross sections evaluated at discrete temperatures, e.g., at 0°C and 21°C, to evaluate cross sections at 10°C, we coded every material to be about half at 0°C and 21°C; this method of temperature correction is well-benchmarked and its implementation explained further in Ref.[9].

6.1 Criticality and Flux

Figure 5 shows the energy spectrum of neutrons in the core with the RaBe source. The RaBe activity is not given, so all results are normalized to be per source neutron. This energy spectrum corroborates the fact that the B8 is a strongly thermal reactor, but with a mode at 0.0533 eV and not the theoretically expected 0.0253 eV.

The criticality of B8 calculated from our fiducial model, i.e., as it was physically built in April 1945, is:

$$\boxed{k_{\text{eff}} = 0.93782 \pm 0.00011}. \tag{4}$$

The percentages of fissions caused by neutrons in the thermal, intermediate, and fast neutron energy ranges are: 87.37% (< 0.625 eV), 4.49% (0.625 eV–100 keV), 8.14% (> 100 keV). The B8 had averages of 2.456 neutrons produced per fission ("fission neutrons"), 1.166 fission neutrons per neutron absorbed in fuel, and 0.9339 fission neutrons per neutron absorbed (radiatively captured + fissioned) in the entire pile assembly.

From the unit cell model, we got $\boxed{k_\infty = 1.11594 \pm 0.00005}$, which retrieves $k_\infty = 1.11$ originally reported by Heisenberg and Wirtz [2; 7], at least validating how we modeled the fuel and moderator. Dividing our k_∞ by k_{eff} gives us a non-leakage factor $\mathcal{L}_{nl}$ of 0.8396. In other words, about 16% of all neutrons are lost by capture in the reflector or physically leaking out the surface of the reactor.

[5]As nuclear physicists and engineers have opposite definitions of absorption and capture, we clarify our convention in this paper: absorption = fission + capture, with absorption corresponding to ENDF reaction number MT 2 and radiative capture corresponding to MT 102 through 117 inclusive. N.B. In Table 5.18 of the MCNP6.3 Manual, special reaction number R -2 is defined as the absorption cross section, but it corresponds to how capture is defined in this paper.

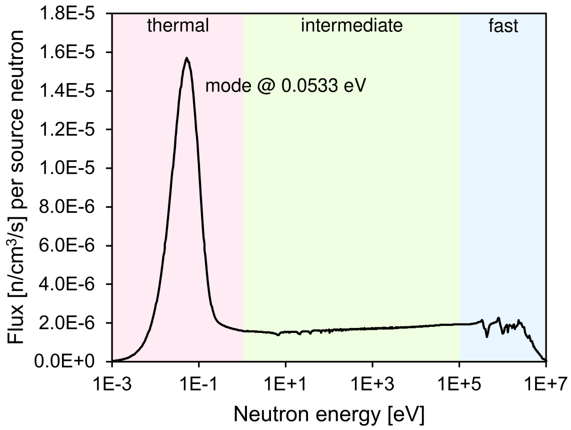


Figure 5: Flux energy spectrum in the core (modr. + fuel) shows the B8 is a thermal reactor with the most probable neutron energy at 0.0533 eV. From MCNP with 10M neutrons, all errors $\pm 1\sigma < 1\%$.

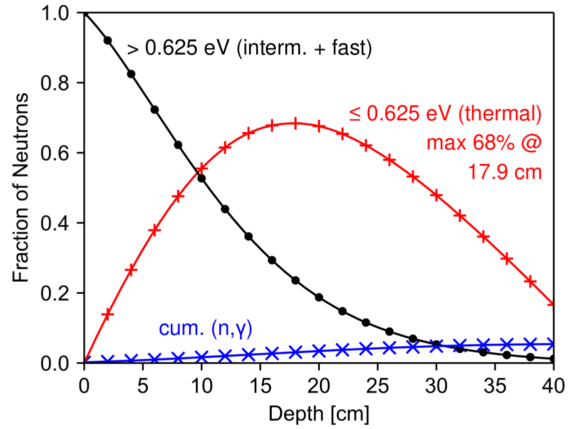


Figure 6: For an initial source of Watt-energy fission neutrons, at most 68% of neutrons become thermal at a depth of 17.876 cm in 96.8 mol% D_2O. From MCNP with 10M neutrons, all errors $\pm 1\sigma < 1\%$.

6.2 Nuclear Lengths

Average pitch, i.e., center-to-center distance between cubes, is 14.89 cm. If we account for cube thicknesses by subtracting the diameter of a sphere of equivalent volume, 6.20 cm, **then the displacement neutrons have to moderate is only 8.69 cm.**

How does this compare to the optimal displacement for thermalization in the B8's 96.8 mol% heavy water? From Fermi age theory, the average displacement of a 1 MeV neutron thermalizing to 1 eV in 96.8 mol% D_2O is:

$$\overline{r} = \left[\frac{2}{\xi \Sigma_s^2 (1 - \overline{\mu})} \ln \left| \frac{E_f}{E_i} \right| \right]^{1/2} = 19.97 \text{ cm}, \tag{5}$$

for cross sections evaluated at the logarithmic midpoint of 1 keV (derivation on GitHub). However, we want to balance this against the fact that heavy water still has a non-zero chance of absorbing neutrons.

We can numerically measure neutron capture in heavy water for continuous-energy cross sections by simulating a sigma pile experiment for B8's heavy water. A sigma pile diffuses neutrons, in our case with a distribution of energies from fission (Watt spectrum), down a long column of the material. We setup this experiment in MCNP and diffuse Watt-energy fission neutrons through a column of the B8's 96.8 mol% pure heavy water. We then measure the neutron current histogram the cumulative capture at various depths. The results are shown in Figure 6. Initially at shallow depths, the total neutron population (sum of all three lines) slightly increases due to > 3 MeV fission neutrons undergoing $(n, 2n)$ reactions. Figure 6 shows that, when accounting for capture, $(n, 2n)$, and elastic + inelastic scattering, **most fission neutrons (68%) reach thermal energies at a depth of 17.876 cm in 96.8 mol% D_2O.**

6.3 Sensitivity Analysis

It is possible that our measurement of fuel cube density at 18.53 g/cm^3 or our 96.8% heavy water purity specification are incorrect. Figure 7 shows a sensitivity analysis of the change in k_{eff} as a function of perturbations in fuel density (orange, top axis) or moderator purity (blue, bottom axis). We see that within the possible ranges of error, **the moderator purity has a much higher percent-change effect on k_{eff} than the fuel density.**

7 Criticality Optimization

7.1 Moderator-to-Fuel Ratio

In Section 6.2 we calculated there was only 8.69 cm of displacement between cube surfaces, when neutrons need 19.97 cm to thermalize in the 96.8 mol% heavy water. This implies the B8 was undermoderated, i.e., had less moderator than optimal, and this magnitude is quantified through the moderator-to-fuel volume ratio. With more moderator molecules relative to fuel, neutrons can more easily slow to thermal energies without being absorbed at resonance energies.

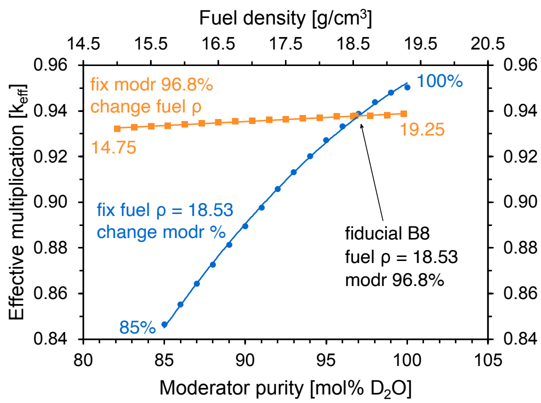


Figure 7: Individual sensitivity analyses of uranium density and moderator purity shows the latter has a much more significant effect on criticality. From MCNP with 200k neutrons/cycle.

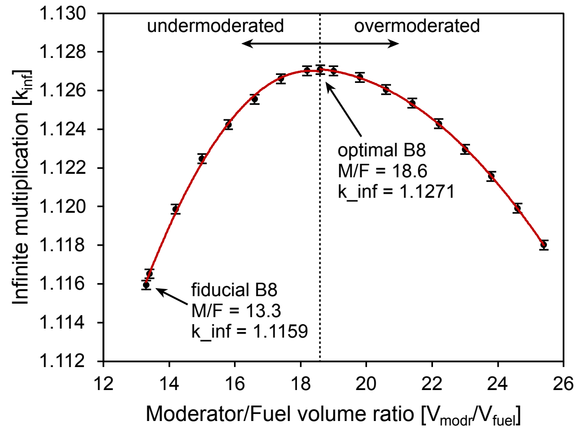


Figure 8: Increasing moderator-to-fuel volume ratio from the fiducial 13.3 to 18.6 shows k_∞ can be increased by 0.0112. From MCNP with 200k neutrons/cycle; error bars $\pm 3\sigma$.

This also decreases migration length, thereby increasing the non-leakage probability. However, if there is too much moderator, thermal neutrons get absorbed by it before it reaches the fuel.

We tested the moderator-to-fuel ratio using the unit cell model drawn in Figure 3. Keeping the single cube (with volume 125 cm^3) fixed, we incrementally increased the volume of the moderator around the cube. Because the unit cell model has reflected boundaries, it simulates having identical cells of fuel above, below, and next to it, and thus by increasing the moderator volume around the cube we are simulating increasing the distancing between cubes in the lattice. Figure 8 shows our results. The fiducial B8, as it was built in 1945, has a M:F of 13.3 and $k_\infty = 1.1159$ as we calculated in Section 6.1. We can see this is quite undermoderated; we reach an optimal moderator-to-fuel ratio of 18.6, i.e., 18.6 times more volume of 96.8 mol% heavy water per 125 cm^3 natural uranium cube. **The fiducial B8 is undermoderated; its fiducial moderator-to-fuel volume ratio is 13.3 but its optimum is 18.6.**

7.2 Theoretical Critical Size

Now that we have an ideal ratio of moderator to fuel, we must calculate the actual physical size of the reactor that achieves the *effective* multiplication factor $k_{\text{eff}} > 1$. To do so, we can solve the criticality condition:

$$BR\cot(BR) = 1 - \frac{D_{\text{refl}}}{D_{\text{core}}}\left(\frac{R}{L_{T,\text{refl}}} + 1\right). \tag{6}$$

Given the moderator-to-fuel ratio, solving for R tells us the radius needed to achieve exactly $k_{\text{eff}} = 1$. Notice this is actually actually for a reflected sphere, as there is no closed-form analytical solution for a totally reflected cylinder like the B8. This is okay, as the critical composition of the equivalent spherical reactor is a very close approximation [8]. Details on computing nuclear constants, like buckling B, diffusion coefficients D_i, and thermal diffusion length L_T, from the results in Section 6 are on the GitHub.

Solving this equation yields the critical radius to be 91.27 cm for a sphere. A square cylinder has optimum dimensions $H = 1.82R$ (c.f. Ref.[8] p.309), so the cylindrical dimensions come out to **radius 82.3 cm and height 149.7 cm.** With a moderator-to-fuel volume ratio of 18.6, this corresponds to **3011 kg of natural uranium (1300 cubes) and 3343 kg of 96.8 mol% pure heavy water.**

7.3 Non-linear Optimization

Now we tackle the non-linear optimization problem of finding the minimum number of natural uranium cubes and volume of 96.8 mol% D_2O moderator to allow a heterogeneous, realistic B8 to go critical. The only way to do this is rather heuristically, by testing various combinations of cubes and moderator and plotting their contours, but thanks to our above calculations, we have narrowed down the critical mix to be about 1300 cubes and 3000 L of moderator, with a moderator to fuel volume ratio of 18.6.

To approach the addition of cubes in a methodical way, we arranged these cubes in a HCP lattice as described in Section 5. Figure 9 shows the resulting contours of criticality for various amounts of cubes and moderator. The Pareto

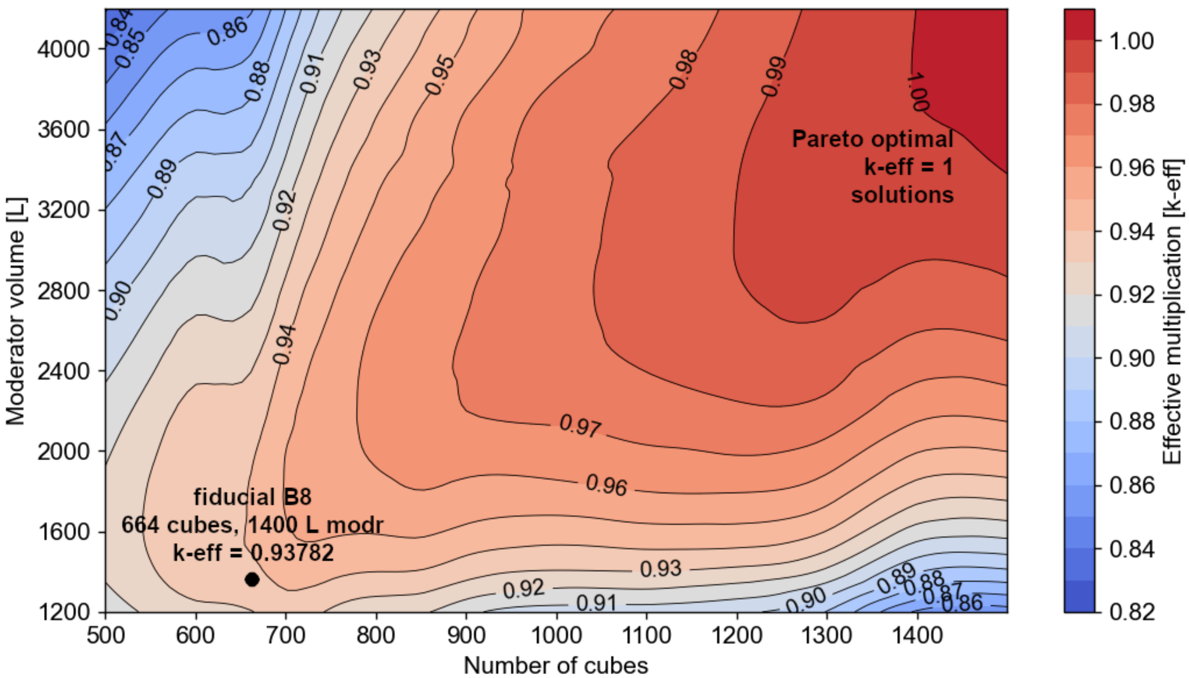


Figure 9: Contour map of the predicted B8 criticality as functions of the number of natural uranium fuel cubes and liters of 96.8 mol% D_2O moderator, cubic-interpolated from criticality calculations of 178 discrete lattice models, each run in MCNP6.2 with 120k neutrons. Criticality $k_{\text{eff}} = 1$ only occurs across a Pareto front of about (1400 cubes + 3600 L) or (1500 cubes + 3390 L of moderator).

optimal front requires at least 1380 cubes and 3700 L of moderator, or 1400 cubes and 3600 L, or 1500 cubes and 3300 L. The contours buckle slightly upward at around 1300 to 1400 cubes because additional cubes require all the more space, i.e., moderator, to thermalize neutrons.

Table 1: Comparison of criticality calculated in this paper, versus by Heisenberg and in prior works, versus the availability of raw materials produced in total by Germany by 1945.

	Heisenberg		Prior works		This paper		Available
	B8 actual	B8 predicted	Grasso et al.	Pešić	B8 actual	B8 predicted	in Germany
k-eff	–	1.00	0.851	0.947	0.938	1.00	–
k-inf	1.11	–	–	–	1.12	1.17	–
N cubes	664	664	664	664	664	1500	2374
V moderator	1400	2100	1400	1400	1400	3390	2643

8 Conclusions and Summary of Results

To revisit our earlier investigative questions, our approach in reconstructing the reactor's specifications and evaluating its criticality highlights the following findings:

1. Section 6 showed that the B8 reactor, as built in April 1945 with 664 cubes and 1400 L heavy water, achieved a criticality of approximately $k_{\text{eff}} = 0.9378$, indicating it was subcritical;
2. Furthermore, the optimization of the moderator-to-fuel ratio revealed that the B8 was significantly undermoderated. By increasing this ratio from the fiducial ratio of 13.3 to an optimal 18.6, we could increase criticality across the board by $\Delta k = 0.0112$; and
3. We know from some archival works, to be published soon, that Nazi Germany only produced a total of 5.5 tons of metallic uranium for reactor experiments and a total of 2,840 kg of 100% D_2O; diluted to 96.8% D_2O at 1.11 g/cm^3 is 2643 L. In Section 7, if we select the Pareto option that minimizes heavy water, then the

B8 would have required 1500 cubes and 3390 L of 96.8 mol% D_2O. This finding discredits Heisenberg's post-war claims that the reactor would have achieved criticality with "a small amount" more uranium and "an increase of the [D_2O] volume by not quite half."

Table 1 compares our results with other contemporary analyses, such as those by Grasso et al. and Pešić. Close agreement with Heisenberg's measured k_∞ and Pešić's calculated k_{eff} seems to validate our modeling approach. Other discrepancies primarily arise from differing assumptions vs. our specifications of material purity and reactor geometry. Our work corrects some of these assumptions, providing a more accurate representation of the B8's performance.

The reconstruction and analysis of the B8, Heisenberg's last reactor experiment, provides a few insights into the technical and scientific challenges faced by the German nuclear program during World War II. The insights gained from this historical reconstruction not only enhance our understanding of early nuclear reactor design but also reinforce the importance of precise material specifications in nuclear archaeology. These lessons remain relevant for modern nuclear forensics and the ongoing efforts to ensure nuclear security and nonproliferation.

Code Availability

All codes (Python scripts, MCNP inputs and outputs) used in this paper are available on a GitHub repository: `https://github.com/patrickpark910/b8heisenberg`. Additional calculation details, omitted from this paper to satisfy Institute for Nuclear Materials Management conference paper page limits, are also available on the repo.

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